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Proof. The result is trivial if $N=0$, so we assume $N\neq 0$. Since the hypotheses and conclusions are invariant under unitary similarity, we may assume without loss that N is diagonal:

$$N=D\oplus 0_m,$$

where D is the nonsingular diagonal matrix:

$$D=e^{i\theta_1}D_1\oplus\cdots\oplus e^{i\theta_k}D_k,$$

where each D_i is an $n_i\times n_i$ real nonsingular diagonal matrix and the θ_i's are distinct numbers satisfying $0\leqslant\theta_i<\pi$. Let $S=PU$, with $P\gg 0$ and U unitary, be the polar decomposition of S. Let Q be the leading $(n_1+\cdots+n_k)\times(n_1+\cdots+n_k)$ principal minor of P^2.

Since we assumed that S^*NS commutes with its adjoint,

$$NP^2N^*=N^*P^2N,$$

from which it follows that

$$D^*QD=DQD^*.$$

Thus Q commutes with $D^*D^{-1}=D^{-1}D^*$. Since

$$D^{-1}D^*=e^{-i2\theta_1}I_{n_1}\oplus\cdots\oplus e^{-i2\theta_k}I_{n_k},$$

and since the numbers $e^{-i2\theta_i}$ are distinct, it follows that

$$Q=Q_1\oplus\cdots\oplus Q_k,$$

where each Q_i is $n_i\times n_i$ and positive definite.

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By Theorem 3.1, $\mathrm{In}(Q_jD_j)=\mathrm{In}(D_j)$. Since Q_jD_j and D_j have real spectrum (Q_jD_j is similar to the Hermitian matrix $\sqrt{Q_j}\,D_j\sqrt{Q_j}$), rotating by $e^{i\theta_j}$ gives

$$\theta\left[Q_j\left(e^{i\theta_j}D_j\right)\right]=\theta\left[e^{i\theta_j}D_j\right].$$

It follows that

$$\begin{aligned}\theta[S^*NS]&=\theta[SS^*N]=\theta[P^2N]=\theta[QD\oplus 0_m]\\&=\theta\left[Q_1\left(e^{i\theta_1}D_1\right)\oplus\cdots\oplus Q_k\left(e^{i\theta_k}D_k\right)\oplus 0_m\right]\\&=\theta\left[\left(e^{i\theta_1}D_1\right)\oplus\cdots\oplus\left(e^{i\theta_k}D_k\right)\oplus 0_m\right]=\theta[N].\end{aligned}$$ ■

This proof characterizes the nonsingular S for which S^*NS is normal.

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